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Bell Inequalities and Separability of Uncommon Causes from the Setup Configuration

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In the present paper the integration region Λ with more than one hidden variable is attributed to a pair of particles in the Bell's thought experiment as the local causal events in their common lightcone. Moreover, the possibility of uncommon causal events influencing the spin measurement is not ignored. Then, with regard to the separability of the influence of the uncommon events from configuration of the setup, and by relying on local realism and coherency, each of the Bell's inequality versions is obtained by measuring spin in three and four different directions.

KEY WORDS: Bell inequalities; hidden variables.

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1. INTRODUCTION

The root of the Bell inequality lies in the well-known paper by Einstein, Podolsky, and Rosen (1935) (EPR). The Bohmian version of EPR thought experiment (EPRB) may so be stated that a particle with total spin zero decays into two particles 1 and 2 which subsequently run away from each other along a straight line. By measuring the spin of particle 1 along some direction, we immediately conclude from spin conservation law (coherency) of particles that the spin of particle 2 is in the opposite direction. According to the idea of local realism it is expected that measuring will not disturb certain properties of events which are sufficiently distant (Clauser and Shimony, 1978). Bell (1964, 1966) highlighted these elements of reality which were hidden from the eyes of quantum mechanics (QM)

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by λ (the hidden variable). Moreover, in a new innovation, he let the measuring angles on the right and on the left be different, and we label them by $\hat{a}$ and $\hat{b}$, respectively. λ symbolizes the realities or events of the common lightcone of the pair of particles which causally affects the spin of both particles, and it may in itself contain more than one variable. In this sense the parameter (or parameters) λ acts as one (or more than one) common label for the pair of particles. From the point of view of hidden variables as such the change in λ means repeating the thought experiment for another pair of particles. Also continuity of λ implies the infinity in number of pair of particles under experience ($N \to \infty$). For this reason, when testing the Bell inequality one considers N to be large. According to the original interpretation of hidden variables theory proposed by Bell λ spans some region Λ representing events of the common lightcones of more than a pair of particles. Hence the spin multiplication correlation of more than one pair of particles is produced. Based on the realism and the idea of locality, Bell proved his famous inequality with three angles $\hat{a}$, $\hat{b}$ and $\hat{b}'$, in terms of three correlation functions which refer to three distinct pairs of configurations. An experimental confirmation of Bell's theorem cannot be drawn from the three angle inequality without making a strong and testable assumption. Two Clauser-Horne-Shimony-Holt (CHSH) inequalities with four angles have also been obtained on the bases of realism and locality assumptions (Bell, 1976; Clauser *et al.*, 1969; Clauser and Shimony, 1978). The CHSH inequalities are expressed in the forms more suitable for performing actual experimental tests. Greenberger, Horne and Zeilinger (later together with Shimony) produced an alternative to the Bell setup, known as the GHZ experiment (Greenberger *et al.*, 1989, 1990).

In general we can assume that the operator corresponding to a given observable may belong to two separate sets of operators such that all elements of each set commute with each other but the elements of the union of the two sets do not commute (Malley, 2004). Therefore, the concept of measurement of an observable is obviously ambiguous, since there can be distinct experimental ways for measuring a single observable. For this reason we encounter the meaning of contextuality which refers to the dependence of measurement resulting on the detailed experimental arrangement being employed. Based on the contextuality, the hidden variable theories should allow for the possibility that different experimental ways for the measurement of an observable might yield different results on an individual system. The theorems of Kochen–Specker (Kochen and Specker, 1967; Peres, 1990, 1991; Smith, 2004) and Mermin (1990, 1993) include examples of observables for which there exist noncompatible measurement methods. The Kochen–Specker theorem shows that any hidden variable theory for quantum measurement must be contextual. Taking these ideas into consideration the Bell inequalities, based on the scenario of uncommon contextual hidden variables (Bell, 1976), have been distracted in two different methods in Fakhri and Taqavi (2005) and now, in this paper, a third method is presented.

2. SEPARABILITY OF UNCOMMON CAUSES FROM THE SETUP CONFIGURATION

Here, as in Fakhri and Taqavi (2005), we consider the realism described by the hidden variable λ in a contextuality which is different from the original interpretation. That is, we suppose the spins of a pair of particles are affected by more than one common cause in the common lightcone. Thus we consider a region Λ of hidden variables λ in the common lightcone as the causal events affecting the spins of the particles 1 and 2. The statistical predictions of QM would be reproduced by calculating appropriate averages over the region Λ of the contextual hidden variables. $\rho(\lambda) \geq 0$ is the weight function corresponding to the effect of the common hidden variables λ on the spins of pair of particles which one may suppose it to be normalized to one in the region Λ:

$$\int_{\Lambda} \rho(\lambda)\, d\lambda = 1. \tag{1}$$

One cannot produce any reason for the non existence of uncommon causes such as η and ζ affecting the spin values of the right and the left particles. η and ζ similar to λ are the elements of reality with the exception that λ affects the spin of both A and B, whereas the events η and ζ affect just A and B, respectively (Bell, 1976). Therefore, it is not appropriate to consider these uncommon causes to be completely nonexistent. Hence, we label the spins of the right and the left particles by η and λ, and ζ and λ, respectively: $A = A(\hat{a}, \eta, \lambda)$ and $B = B(\hat{b}, \zeta, \lambda)$. Thus what we measure are $A(\hat{a}, \eta, \lambda)$ and $B(\hat{b}, \zeta, \lambda)$, but we cannot understand in detail the effect of common variables λ as well as the uncommon factors η and ζ on the spin of the particles. In the original interpretation λ was the label for a pair of particles, whereas in the method of thinking of the present paper and Fakhri and Taqavi (2005) Λ is the label for a pair of particles. Previously the Bell inequalities were extracted for innumerable particle pairs whereas in the present method one extracts the Bell inequalities for a pair of particles. It is exactly for this reason that we can bring in the uncommon factors η and ζ affecting the spins of particles 1 and 2 respectively.

Now we define the correlation functions for the spin production of particles 1 and 2 in five different setups along $\hat{a}$ and $\hat{b}$, $\hat{a}$ and $\hat{b}'$, $\hat{b}$ and $\hat{b}'$, $\hat{a}'$ and $\hat{b}$, $\hat{a}'$ and $\hat{b}'$, as follows:

$$C_{\eta\zeta}(\hat{a}, \hat{b}) = \int_{\Lambda} A(\hat{a}, \eta, \lambda) B(\hat{b}, \zeta, \lambda)\, \rho(\lambda)\, d\lambda \tag{2a}$$

$$C_{\eta'\zeta'}(\hat{a}, \hat{b}') = \int_{\Lambda} A(\hat{a}, \eta', \lambda) B(\hat{b}', \zeta', \lambda)\, \rho(\lambda)\, d\lambda \tag{2b}$$

$$C_{\eta''\zeta''}(\hat{b},\hat{b}') = \int_\Lambda A(\hat{b},\eta'',\lambda)B(\hat{b}',\zeta'',\lambda)\,\rho(\lambda)\,d\lambda \tag{2c}$$

$$C_{\eta''\zeta''}(\hat{a}',\hat{b}) = \int_\Lambda A(\hat{a}',\eta'',\lambda)B(\hat{b},\zeta'',\lambda)\,\rho(\lambda)\,d\lambda \tag{2d}$$

$$C_{\eta'''\zeta'''}(\hat{a}',\hat{b}') = \int_\Lambda A(\hat{a}',\eta''',\lambda)B(\hat{b}',\zeta''',\lambda)\,\rho(\lambda)\,d\lambda\,. \tag{2e}$$

In Eqs. (2) the correlation of spin production of left and right particle pair is realized through the set of hidden variables Λ as their common causes, whereas in the original interpretation the spin correlation was related to countless particle pairs through different λ belonging to Λ. Another point worth mentioning is the use of a common region Λ for calculating any of the Eqs. (2). This means by changing the configurations for the particle pair under discussion, the common hidden affecting factors Λ must not change. But different subindices in the equations imply that the uncommon factor can change from one thought experiment to another. Since the values of A and B, in terms of $\frac{\hbar}{2}$, are either $+1$ or -1, which describe the spins of particles 1 and 2 in different directions, it is expected that they have the following mathematical properties:

$$\begin{aligned} |A(\hat{a},\eta,\lambda)| &= \left|A(\hat{a},\eta',\lambda)\right| = \left|A(\hat{b},\eta'',\lambda)\right| \qquad (3)\\ &= \left|B(\hat{b},\zeta,\lambda)\right| = \left|B(\hat{b}',\zeta',\lambda)\right| = \cdots = 1 \end{aligned}$$

$$\begin{aligned} A(\hat{b},\eta,\lambda)B(\hat{b},\zeta,\lambda) &= A(\hat{b}',\eta'',\lambda)B(\hat{b}',\zeta'',\lambda) \qquad (4)\\ &= A(\hat{a},\eta',\lambda)B(\hat{a},\zeta',\lambda) = \cdots = -1\,. \end{aligned}$$

Equation (4) describe coherency of particles 1 and 2 on the both sides. According to Eq. (3), since spin only accepts values $+1$ or -1 thus if the factors η, ζ, ... as well as $\hat{a}$, $\hat{b}$, ... could change the spin values, this change would show up as a multiple of $+1$ or -1. *This means that the terms $A(\hat{a},\eta,\lambda)/A(\hat{a},\zeta,\lambda)$ and $A(\hat{a},\eta,\lambda)/A(\hat{b},\eta,\lambda)$ as such, with the allowed values $+1$ or -1, must be independent from $\hat{a}$ and η respectively, because any change in $\hat{a}$ (or η) leading to a sign change in $A(\hat{a},\eta,\lambda)$ will have the same effect on $A(\hat{a},\zeta,\lambda)$ (or $A(\hat{b},\eta,\lambda)$) as well.* We can explain the mathematical concept of change in sign of spin values due to the aforementioned factors in the following way:

$$A(\hat{a},\eta,\lambda) = (\text{a term independent from } \hat{a} \text{ but a function of } \eta \text{ and } \zeta)$$
$$A(\hat{a},\zeta,\lambda)$$

$$A(\hat{a},\eta,\lambda) = (\text{a term independent from } \eta \text{ but a function of } \hat{a} \text{ and } \hat{b})$$
$$A(\hat{b},\eta,\lambda)\,. \tag{5}$$

Therefore, although the functionality of spin values of particles 1 and 2, i.e. A and B, from their arguments are unclear to us, we can accept the following decompositions:

$$\begin{aligned} &A(\hat{a},\eta,\lambda) = f_A(\hat{a},\lambda)g_A(\eta,\lambda) \qquad B(\hat{b},\zeta,\lambda) = f_B(\hat{b},\lambda)g_B(\zeta,\lambda)\\ &A(\hat{a},\eta',\lambda) = f_A(\hat{a},\lambda)g_A(\eta',\lambda) \qquad B(\hat{b}',\zeta',\lambda) = f_B(\hat{b}',\lambda)g_B(\zeta',\lambda)\\ &(\text{and other similar equations}). \end{aligned} \tag{6}$$

Thus, resorting to the independency of uncommon events η and ζ from the chosen angles $\hat{a}$ and $\hat{b}$ for measuring the spins of particles along their directions, also the fact that each of these factors, i.e. f_A, g_A, f_B and g_B, can only play the role of change in the sign of the spin values as a factor, brings in the possibility of decomposing spins functionality into the product of the factors which include these events and angles. Equation (6), with regard to the relations (3) and (4), give

$$|f_A(\hat{a},\lambda)|\,|g_A(\eta,\lambda)| = \left|f_B(\hat{b},\lambda)\right|\,|g_B(\zeta,\lambda)| = \cdots = 1 \tag{7a}$$

$$\begin{aligned} f_A(\hat{a},\lambda)g_A(\eta,\lambda)f_B(\hat{a},\lambda)g_B(\zeta,\lambda) &= f_A(\hat{b},\lambda)g_A(\eta,\lambda)f_B(\hat{b},\lambda)g_B(\zeta,\lambda)\\ &= \cdots = -1\,. \end{aligned} \tag{7b}$$

The fulfilment of Eq. (7a) does not necessarily require that f_A and g_B be ± 1. But if the values ± 1 are chosen for f_A and g_B, then these choices must be such that the Eq. (7b) are also satisfied. Notice that the functionality of the factors f_A and g_B of the hidden variables λ as the common causal events is immaterial, specially that in calculating the correlation functions one integrates over λ, and λ's (or Λ) in general are unknown. But the events $\hat{a}$, $\hat{b}$, η and ζ have a completely different situation from λ in that they can be known and accessible events. However, whether or not one or both of the factors f_A and g_A (f_B and g_B) are some function of λ, it has no effect in deriving and proving the Bell inequalities in the following section.

3. COHERENCY AND LOCAL REALISM TOWARDS BELL INEQUALITIES

Now we are in a position to derive three and four angle Bell inequalities for the given correlation functions in Eqs. (2) by using the idea of separability of the influences of uncommon causes η, ζ, ... from the setup configuration $\hat{a}$, $\hat{b}$, ... as in the Eq. (6). First we investigate the Bell inequality with three angles. From

Eqs. (2a) and (2b) we conclude that

$$C_{\eta\zeta}(\hat{a},\hat{b}) - C_{\eta'\zeta'}(\hat{a},\hat{b}') = \int_\Lambda A(\hat{a},\eta,\lambda)B(\hat{b},\zeta,\lambda) \left[1 - \frac{A(\hat{a},\eta',\lambda)B(\hat{b}',\zeta',\lambda)}{A(\hat{a},\eta,\lambda)B(\hat{b},\zeta,\lambda)}\right]\rho(\lambda)\,d\lambda. \tag{8}$$

With regard to the condition $\rho(\lambda) \geq 0$ and taking Eq. (3) into consideration, also by calculating the modulus of both sides of (8) we get

$$\left|C_{\eta\zeta}(\hat{a},\hat{b}) - C_{\eta'\zeta'}(\hat{a},\hat{b}')\right| \leq \int_\Lambda \left|1 - \frac{A(\hat{a},\eta',\lambda)B(\hat{b}',\zeta',\lambda)}{A(\hat{a},\eta,\lambda)B(\hat{b},\zeta,\lambda)}\right|\rho(\lambda)\,d\lambda\,. \tag{9}$$

Repetitive application of Eqs. (4) and (6) gives

$$\begin{aligned}
\left|C_{\eta\zeta}(\hat{a},\hat{b}) - C_{\eta'\zeta'}(\hat{a},\hat{b}')\right| &\leq \int_\Lambda \left|1 - \frac{f_A(\hat{a},\lambda)g_A(\eta',\lambda)B(\hat{b}',\zeta',\lambda)}{f_A(\hat{a},\lambda)g_A(\eta,\lambda)B(\hat{b},\zeta,\lambda)}\right|\rho(\lambda)\,d\lambda \\
&= \int_\Lambda \left|1 - \frac{f_A(\hat{b},\lambda)g_A(\eta',\lambda)B(\hat{b}',\zeta',\lambda)}{f_A(\hat{b},\lambda)g_A(\eta,\lambda)B(\hat{b},\zeta,\lambda)}\right|\rho(\lambda)\,d\lambda \\
&= \int_\Lambda \left|1 - \frac{A(\hat{b},\eta',\lambda)B(\hat{b}',\zeta',\lambda)}{A(\hat{b}',\eta',\lambda)B(\hat{b}',\zeta',\lambda)}\right|\rho(\lambda)\,d\lambda \\
&= \int_\Lambda \left|1 - \frac{f_A(\hat{b},\lambda)g_A(\eta'',\lambda)B(\hat{b}',\zeta'',\lambda)}{f_A(\hat{b}',\lambda)g_A(\eta'',\lambda)B(\hat{b}',\zeta'',\lambda)}\right|\rho(\lambda)\,d\lambda \\
&= \int_\Lambda \left|1 + A(\hat{b},\eta'',\lambda)B(\hat{b}',\zeta'',\lambda)\right|\rho(\lambda)\,d\lambda.
\end{aligned} \tag{10}$$

Now, let us consider $A(\hat{b},\eta'',\lambda)B(\hat{b}',\zeta'',\lambda) \geq -1$ and use Eqs. (1) and (2c) in the relation (10) to obtain

$$\left|C_{\eta\zeta}(\hat{a},\hat{b}) - C_{\eta'\zeta'}(\hat{a},\hat{b}')\right| \leq 1 + C_{\eta''\zeta''}(\hat{b},\hat{b}')\,. \tag{11}$$

The relation (11) is a statement of Bell's inequality corresponding to the correlation functions of a pair of particles. These correlation functions correspond to three different configurations $\hat{a}$ and $\hat{b}$, $\hat{a}$ and $\hat{b}'$, $\hat{b}$ and $\hat{b}'$ affected by the events η and ζ, η' and ζ', η'' and ζ'', respectively.

Similarly we can obtain the four angle inequalities for the correlation functions. To this end, first we note that letting

$$\mathcal{A}(\lambda) := A(\hat{a},\eta,\lambda)B(\hat{b},\zeta,\lambda)A(\hat{a}',\eta''',\lambda)B(\hat{b}',\zeta''',\lambda) \tag{12}$$

we have

$$
\begin{aligned}
0 &= \pm \int_\Lambda \rho(\lambda) d\lambda \, (1-1)\, \mathcal{A}(\lambda) \\
&= \pm \int_\Lambda \rho(\lambda) d\lambda \left(1 - \frac{A(\hat{a},\eta,\lambda) f_B(\hat{a},\lambda) g_B(\zeta,\lambda) A(\hat{b},\eta''',\lambda) f_B(\hat{b},\lambda) g_B(\zeta''',\lambda)}{A(\hat{a},\eta,\lambda) f_B(\hat{b},\lambda) g_B(\zeta,\lambda) A(\hat{b},\eta''',\lambda) f_B(\hat{a},\lambda) g_B(\zeta''',\lambda)}\right) \mathcal{A}(\lambda) \\
&= \pm \int_\Lambda \rho(\lambda) d\lambda \left(1 - \frac{A(\hat{a},\eta,\lambda) B(\hat{a},\zeta,\lambda) A(\hat{b},\eta''',\lambda) B(\hat{b},\zeta''',\lambda)}{A(\hat{a},\eta,\lambda) B(\hat{b},\zeta,\lambda) A(\hat{b},\eta''',\lambda) B(\hat{a},\zeta''',\lambda)}\right) \mathcal{A}(\lambda) \\
&= \pm \int_\Lambda \rho(\lambda) d\lambda \left(1 - \frac{A(\hat{a},\eta',\lambda) B(\hat{a},\zeta',\lambda) A(\hat{b},\eta'',\lambda) B(\hat{b},\zeta'',\lambda)}{A(\hat{a},\eta,\lambda) B(\hat{b},\zeta,\lambda) A(\hat{b},\eta''',\lambda) B(\hat{a},\zeta''',\lambda)}\right) \mathcal{A}(\lambda) \\
&= \pm \int_\Lambda \rho(\lambda) d\lambda \left(1 - \frac{A(\hat{a},\eta',\lambda) f_B(\hat{a},\lambda) g_B(\zeta',\lambda) f_A(\hat{b},\lambda) g_A(\eta'',\lambda) B(\hat{b},\zeta'',\lambda)}{A(\hat{a},\eta,\lambda) B(\hat{b},\zeta,\lambda) f_A(\hat{b},\lambda) g_A(\eta''',\lambda) f_B(\hat{a},\lambda) g_B(\zeta''',\lambda)}\right) \mathcal{A}(\lambda) \\
&= \pm \int_\Lambda \rho(\lambda) d\lambda \left(1 - \frac{A(\hat{a},\eta',\lambda) f_B(\hat{b}',\lambda) g_B(\zeta',\lambda) f_A(\hat{a}',\lambda) g_A(\eta'',\lambda) B(\hat{b},\zeta'',\lambda)}{A(\hat{a},\eta,\lambda) B(\hat{b},\zeta,\lambda) f_A(\hat{a}',\lambda) g_A(\eta''',\lambda) f_B(\hat{b}',\lambda) g_B(\zeta''',\lambda)}\right) \mathcal{A}(\lambda) \\
&= \pm \int_\Lambda \rho(\lambda) d\lambda \big(A(\hat{a},\eta,\lambda) B(\hat{b},\zeta,\lambda) A(\hat{a}',\eta''',\lambda) B(\hat{b}',\zeta''',\lambda) \\
&\quad - A(\hat{a},\eta',\lambda) B(\hat{b}',\zeta',\lambda) A(\hat{a}',\eta'',\lambda) B(\hat{b},\zeta'',\lambda)\big)\,. \qquad (13)
\end{aligned}
$$

Applying the result (13) we get

$$
\begin{aligned}
&C_{\eta\zeta}(\hat{a},\hat{b}) - C_{\eta'\zeta'}(\hat{a},\hat{b}') = \int_\Lambda \rho(\lambda) d\lambda A(\hat{a},\eta,\lambda) B(\hat{b},\zeta,\lambda) \\
&\big(1 \pm A(\hat{a}',\eta''',\lambda) B(\hat{b}',\zeta''',\lambda)\big) \\
&- \int_\Lambda \rho(\lambda) d\lambda A(\hat{a},\eta',\lambda) B(\hat{b}',\zeta',\lambda) \big(1 \pm A(\hat{a}',\eta'',\lambda) B(\hat{b},\zeta'',\lambda)\big)\,. \qquad (14)
\end{aligned}
$$

Now by taking modulus of both sides of the Eq. (14) we get at once

$$
\left|C_{\eta\zeta}(\hat{a},\hat{b}) - C_{\eta'\zeta'}(\hat{a},\hat{b}')\right| \le 2 \pm C_{\eta''\zeta''}(\hat{a}',\hat{b}) \pm C_{\eta'''\zeta'''}(\hat{a}',\hat{b}')\,. \qquad (15)
$$

The relation (15) is a fundemental four angle inequality for correlation functions, from which we can derive, for example, the other forms of four angle CHSH inequalities:

$$
\left|C_{\eta\zeta}(\hat{a},\hat{b}) - C_{\eta'\zeta'}(\hat{a},\hat{b}')\right| + \left|C_{\eta''\zeta''}(\hat{a}',\hat{b}) + C_{\eta'''\zeta'''}(\hat{a}',\hat{b}')\right| \le 2 \qquad (16a)
$$

$$
\left|C_{\eta\zeta}(\hat{a},\hat{b}) - C_{\eta'\zeta'}(\hat{a},\hat{b}') + C_{\eta''\zeta''}(\hat{a}',\hat{b}) + C_{\eta'''\zeta'''}(\hat{a}',\hat{b}')\right| \le 2\,. \qquad (16b)
$$

Since the subindices in the relations (11), (16a) and (16b) are free, they can be considered as an excellent symmetry for the Bell inequalities. One can, therefore, with negligence disregard the subindices. However, note that not writing them does not imply their nonexistence. Such kind of interpretation avails the possibility of talking about average correlation functions for N pair of particles in each setup, and neglects the effect of the events η, ζ, ... on the average correlation functions. It appears that there is no need the number of particle pairs be innumerable, and the inequalities also hold for one or finite number of particle pairs. To increase the number of particle pairs to test the Bell inequalities will merely result in sharing the measurement error.

4. CONCLUSION

In this paper some extensions of the standard Bell inequalities based on new assumptions about the nature of the hidden variables are presented. In addition to the usual hidden variables that depend on events in the common lightcone of the two correlated particles, it is proposed to consider hidden variables that also lie in the individual lightcones of each of the particles. With the assumption that these individual hidden variables (uncommon causal events) are uncorrelated from the directions along which spin measurements are made, the modified inequalities are derived. By attributing all hidden variables Λ (but not just one variable λ) to a pair of particles as their common ID, we were able not to necessarily neglect the effect of uncommon causal events over the spins of particles 1 and 2. For the reason that in the previous attribution λ was allocated to a pair of particles, thus Λ to more than a pair of particles, such an attribution was not possible. Since the effect of uncommon factors η, ζ, ... over the spin values is independent of the choice of the prolongations $\hat{a}$, $\hat{b}$, ... in the setup, therefore their effects over the spin values are naturally separable. Taking this fact into consideration, also accepting the principle of the local realism where λ represents it, and further considering the coherency of particles, different versions of Bell inequalities are derived. This method, based on the separability of uncommon events from the setup configurations selection, is substituted by the idea of function u which establishes an equivalence relation amongst all setup configurations (Fakhri and Taqavi, 2005).

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